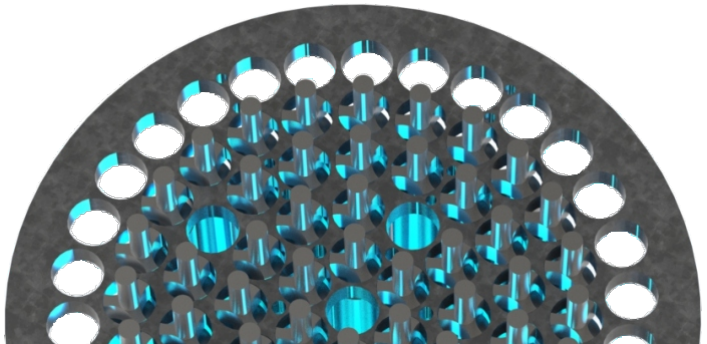
Institut ”Jožef Stefan”
Odsek za reaktorsko fiziko

2. Konferenca Mladih Odseka F8

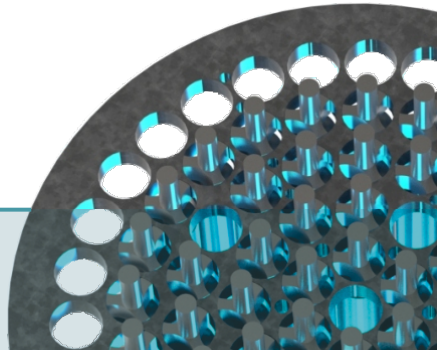
Homogenost sevalnega polja v okolici gorivnih elementov

Junoš Lukan



Reaktorski center
Podgorica, 19. 2. 2015

Vsebina



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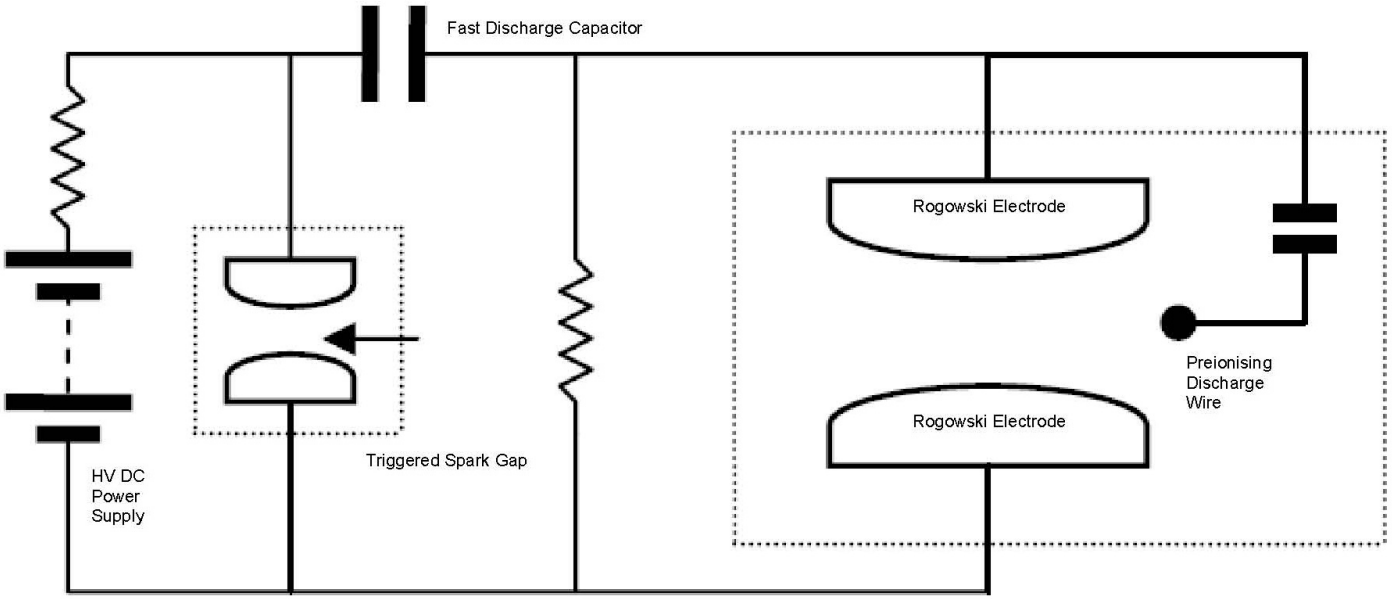

Slika: Maggs, P., 2008, CO2 TEA Laser Electrical Circuit, http://commons.wikimedia.org/wiki/File:TEA-Laser-Circuit.jpg

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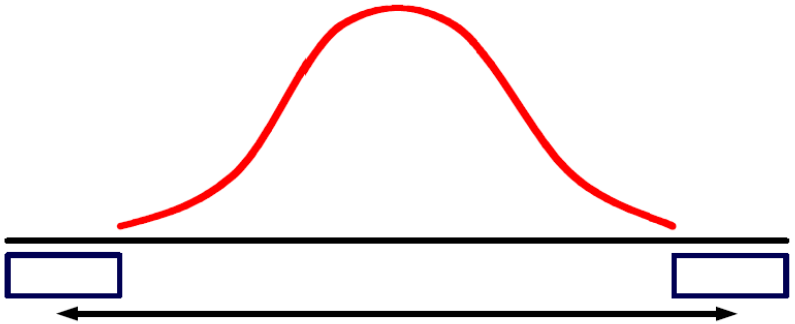
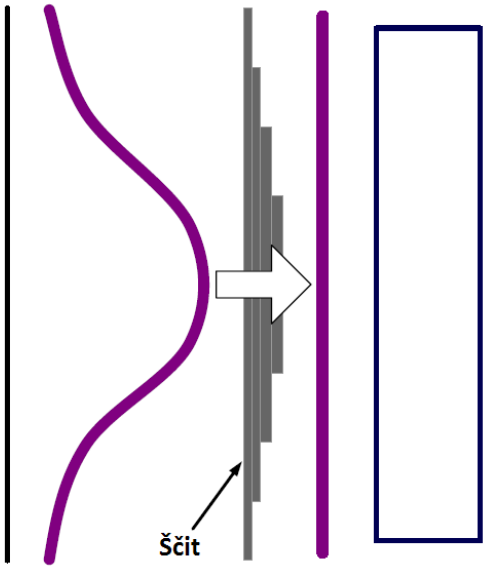


Slika: Premikanje vzorca v radialni smeri. ©IAEA, 2012, slika 9, str. 35[1]



Slika: Premikanje vzorca v aksialni smeri. ©IAEA, 2012, slika 11, str. 37[1]

1 http://www-pub.iaea.org/books/IAEABooks/8739/Neutron-Transmutation-Doping-of-Silicon-at-Research-Reactors-at-Research-Reactors

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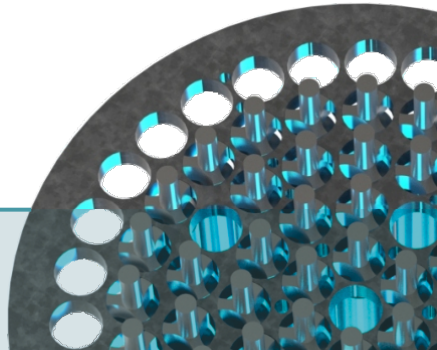
- Dopiranje kristalov v raziskovalnih reaktorjih (*Neutron Transmutation Doping of Silicon at Research Reactors*, 2012)
- Enakomerno obsevanje vzorcev v polju gama za testiranje vzdržljivosti



Analitične definicije

- Naključno polje: $\xi(g) = \sum_{\lambda} \sum_{i,j=1}^{d_\lambda} z_{ji}^{(\lambda)} T_{ij}^{(\lambda)}(g)$, kjer so $z_{ji}^{(\lambda)} = d_\lambda \int_G \xi(g) \overline{T_{ij}^{(\lambda)}(g)} \, \mathrm{d}g$, je homogeno (Yaglom, 1961), če:

$$E z_{ji}^{(\lambda)} \overline{z_{ik}^{(\mu)}} = \delta_{\lambda\mu} \delta_{ik} f_{jl}^{(\lambda)}$$

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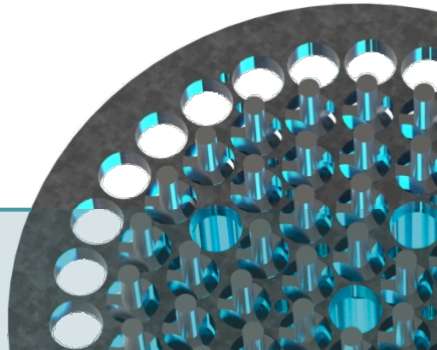

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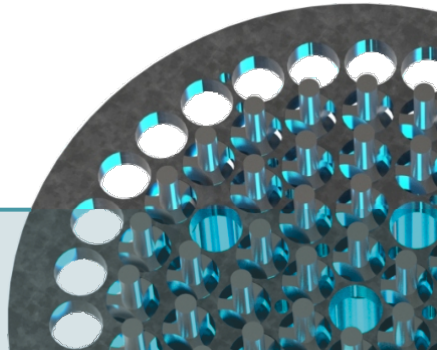
- Delovna definicija: Čim bolj konstantno polje



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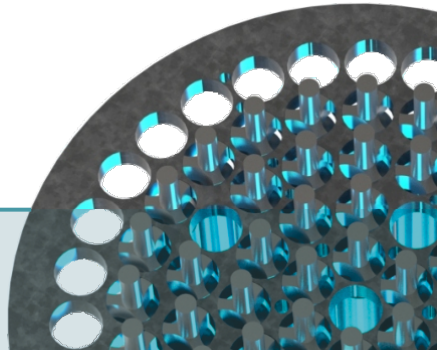

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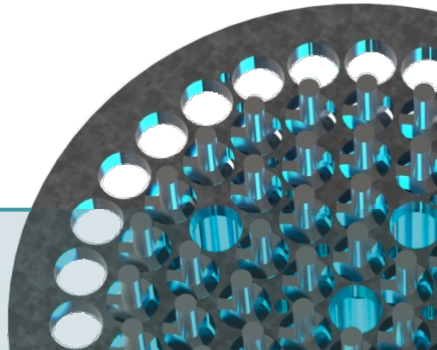

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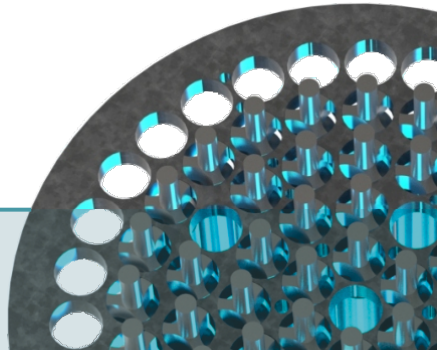

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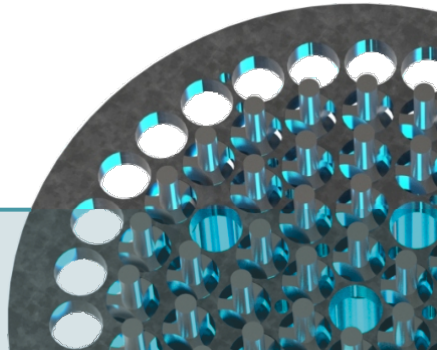

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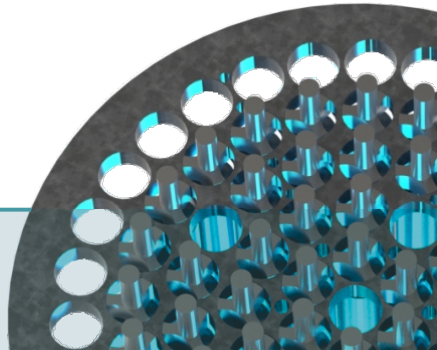

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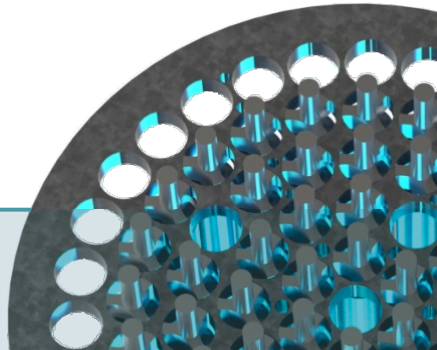

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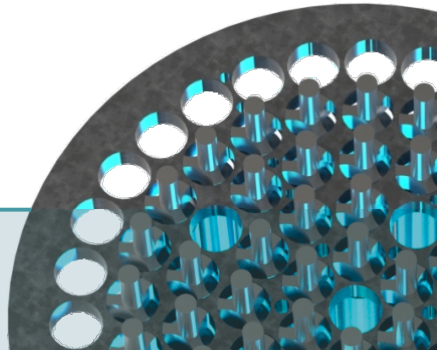

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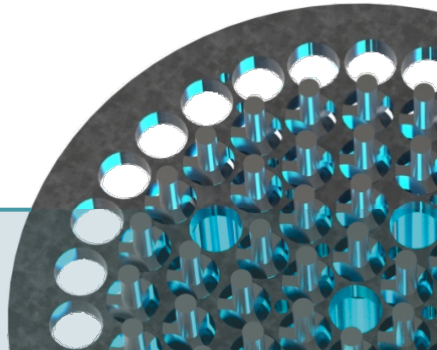

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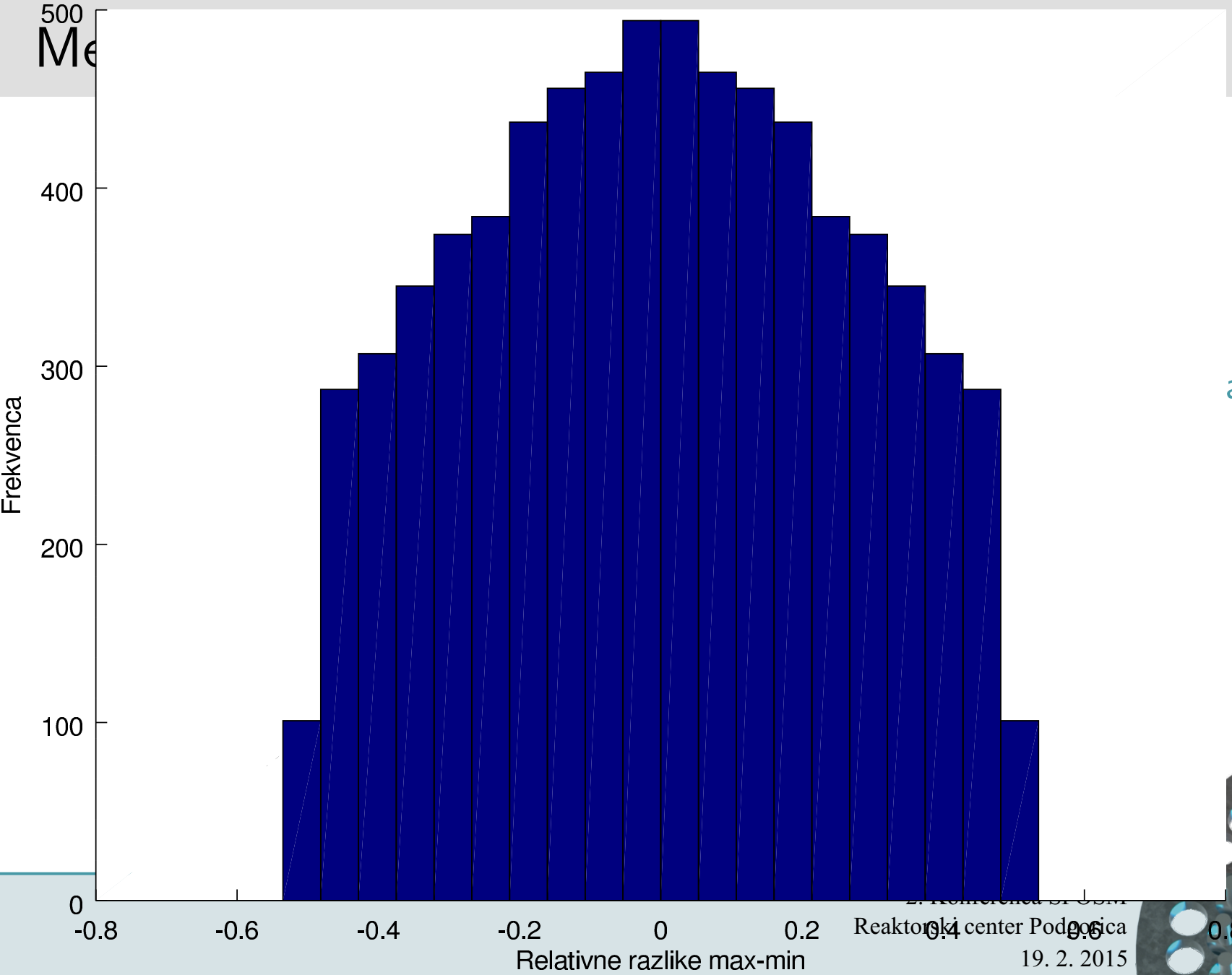
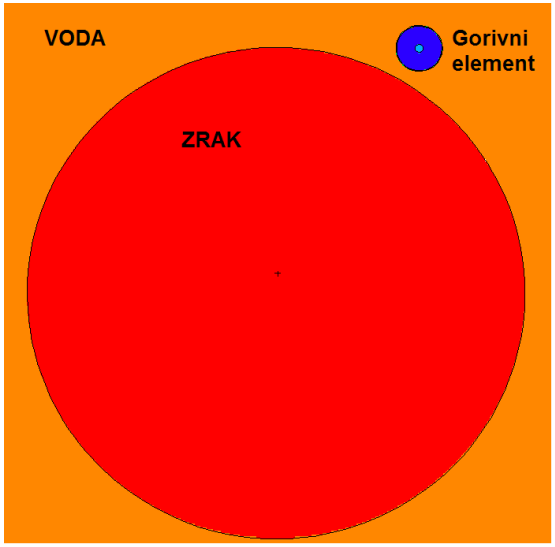

500
400
300
200
100
0
Frekvenca
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Relativne razlike max-min

antno

Nesimetrična razporeditev



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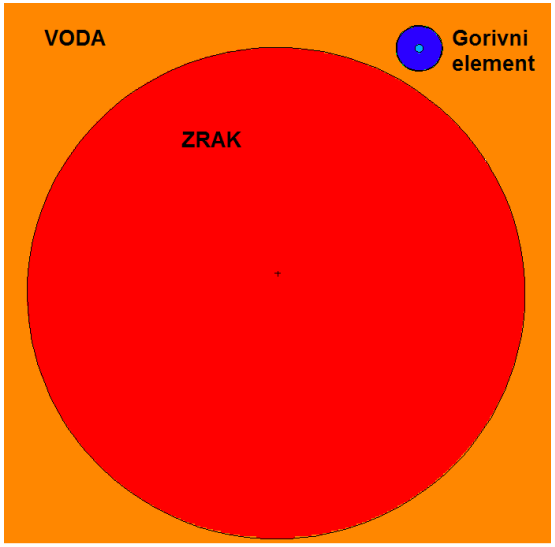

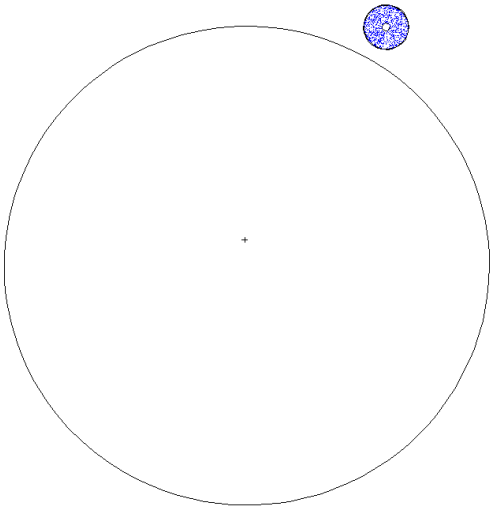

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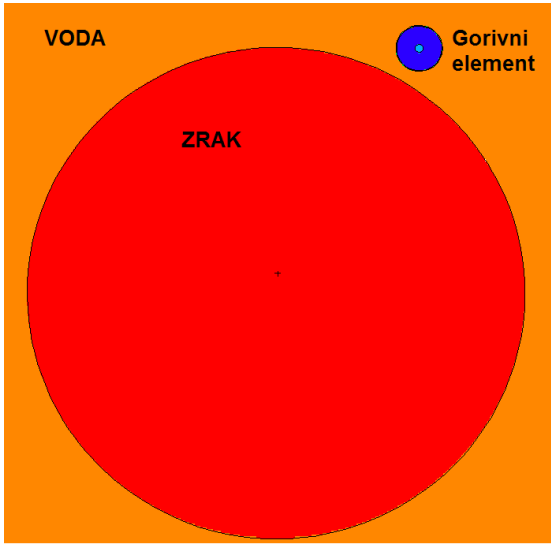

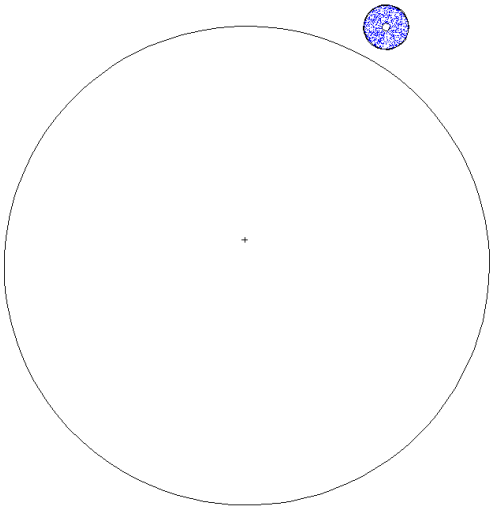

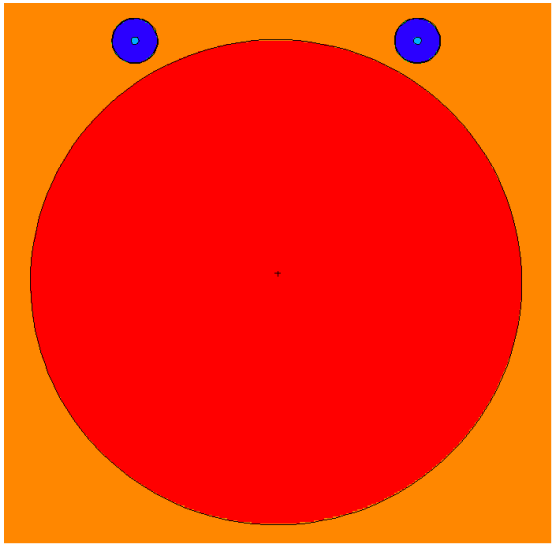

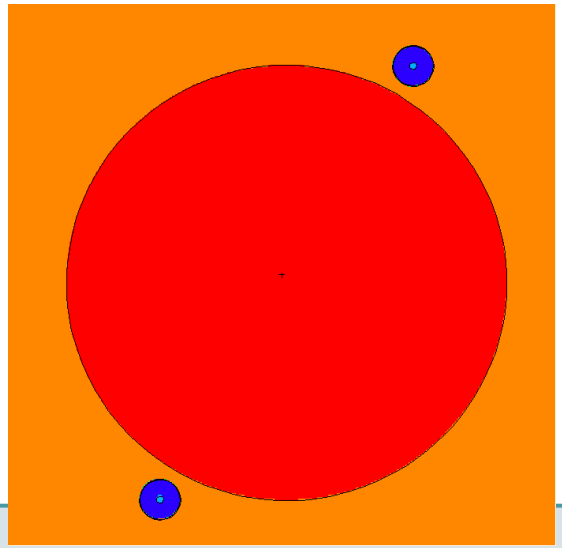

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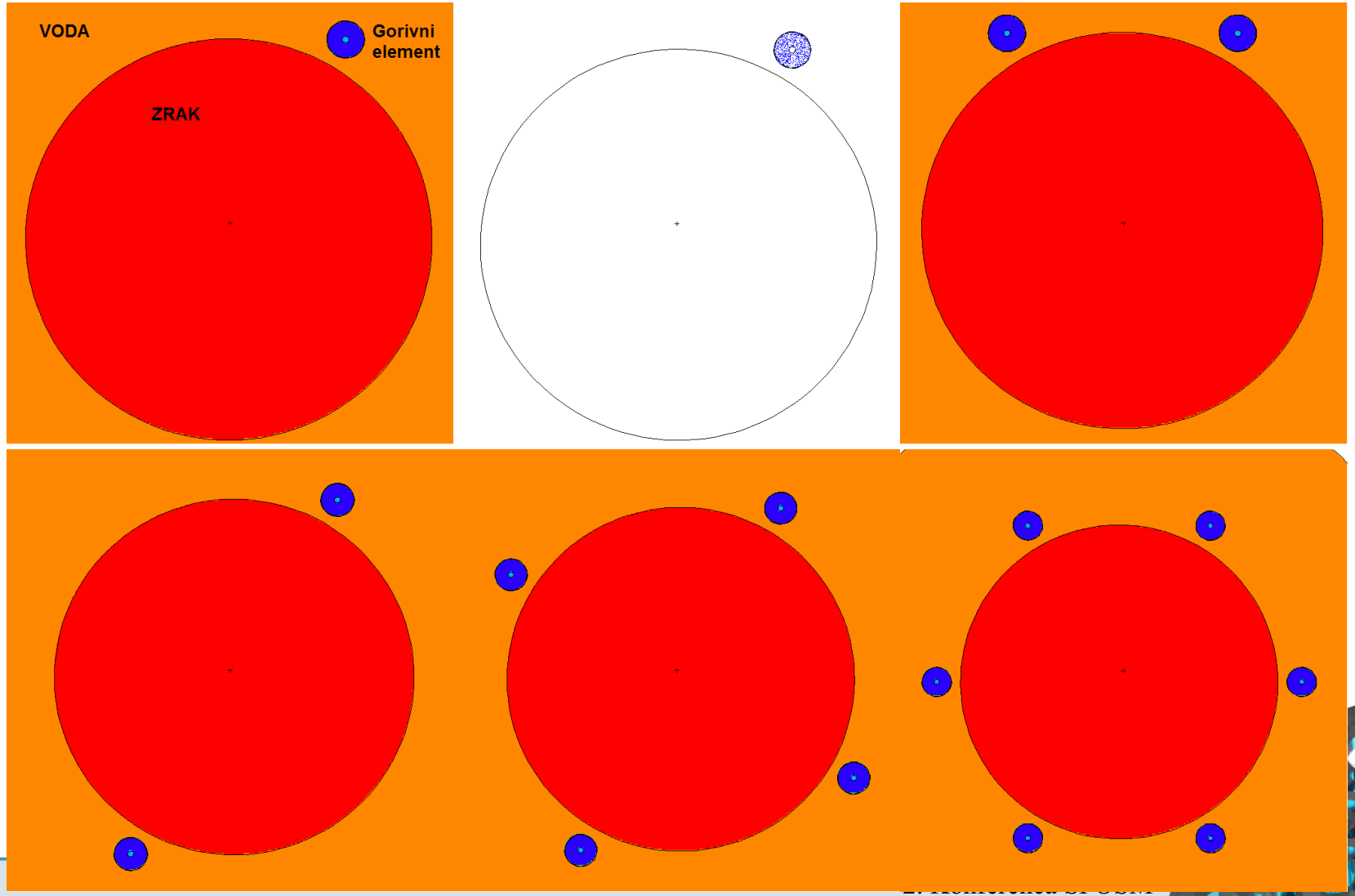
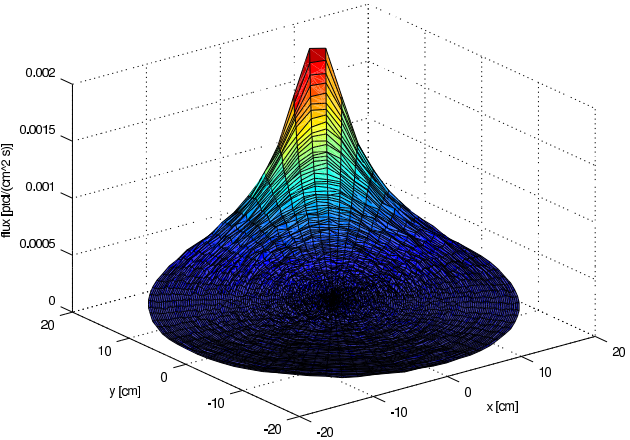
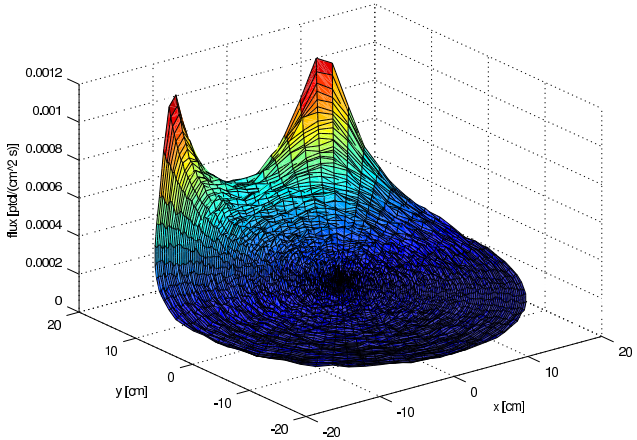
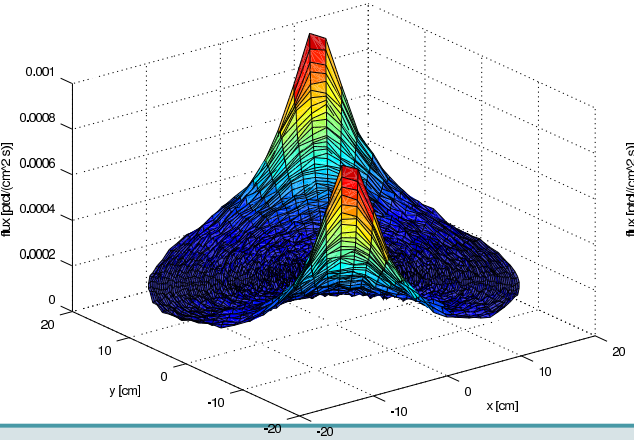
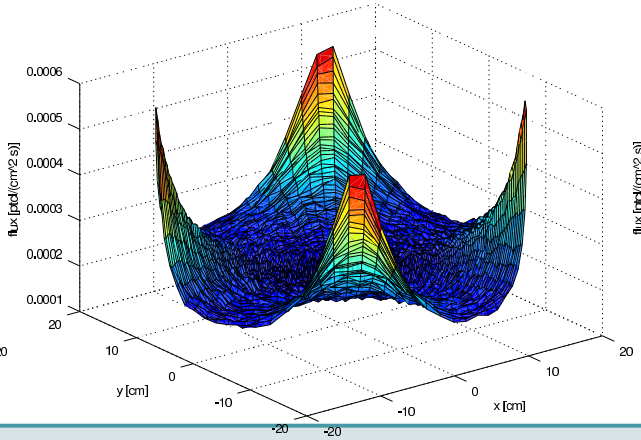


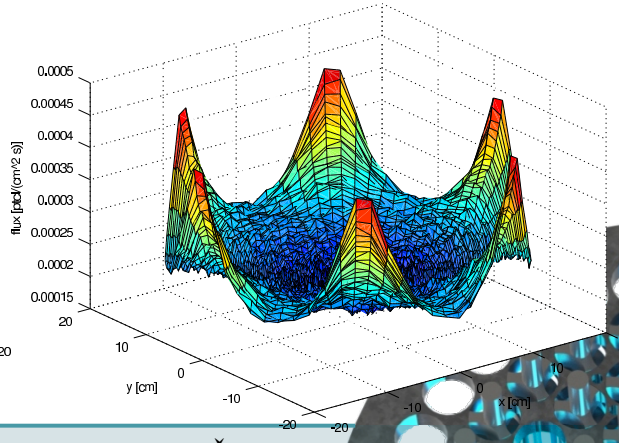
Nesimetrična razporeditev – fluks žarkov gama



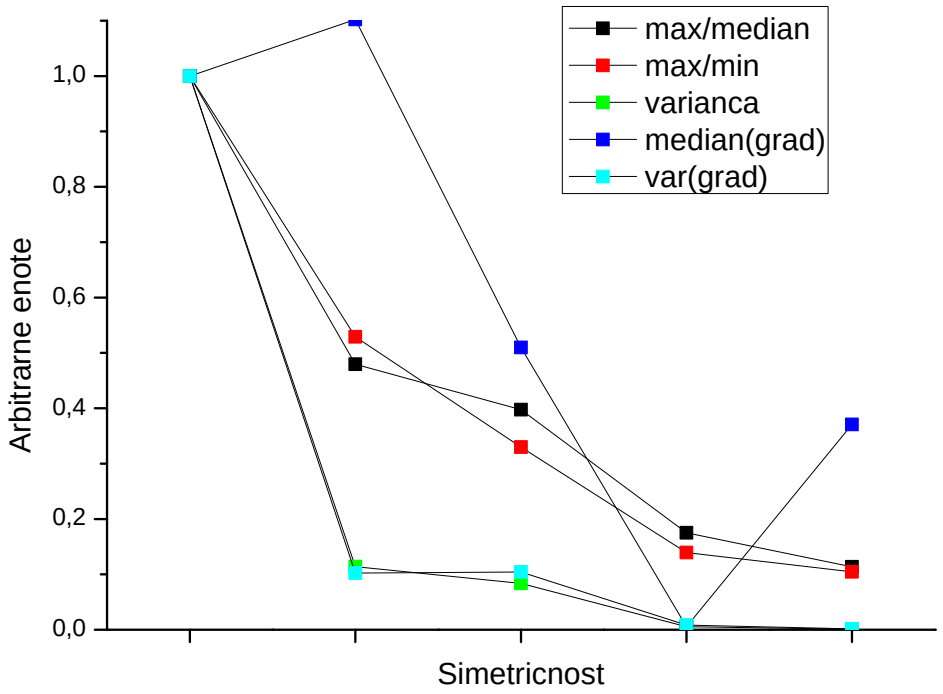


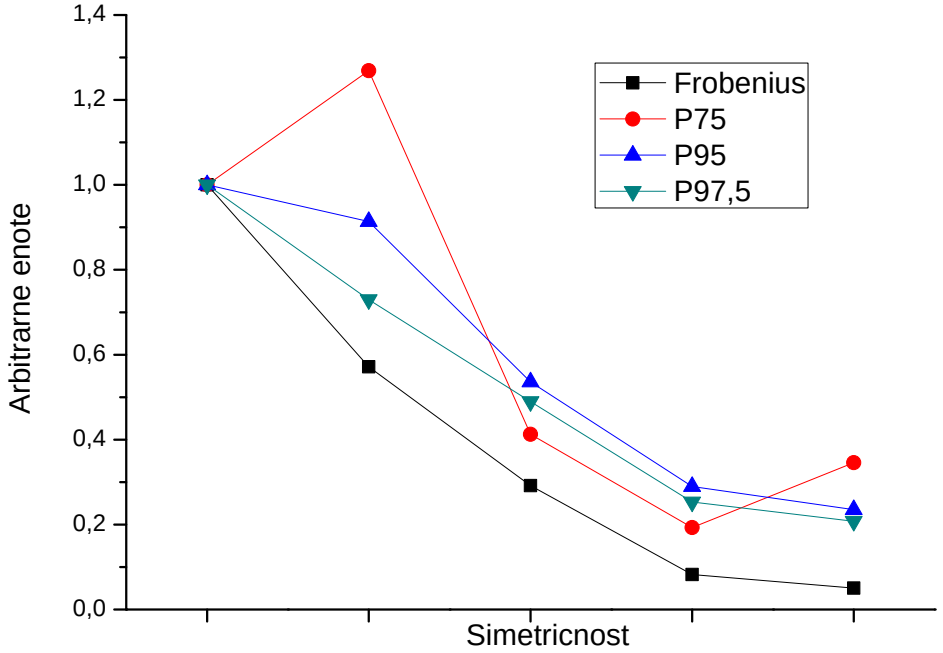


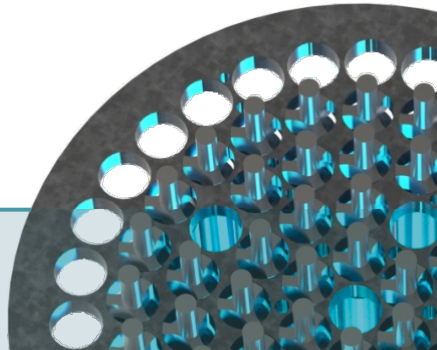




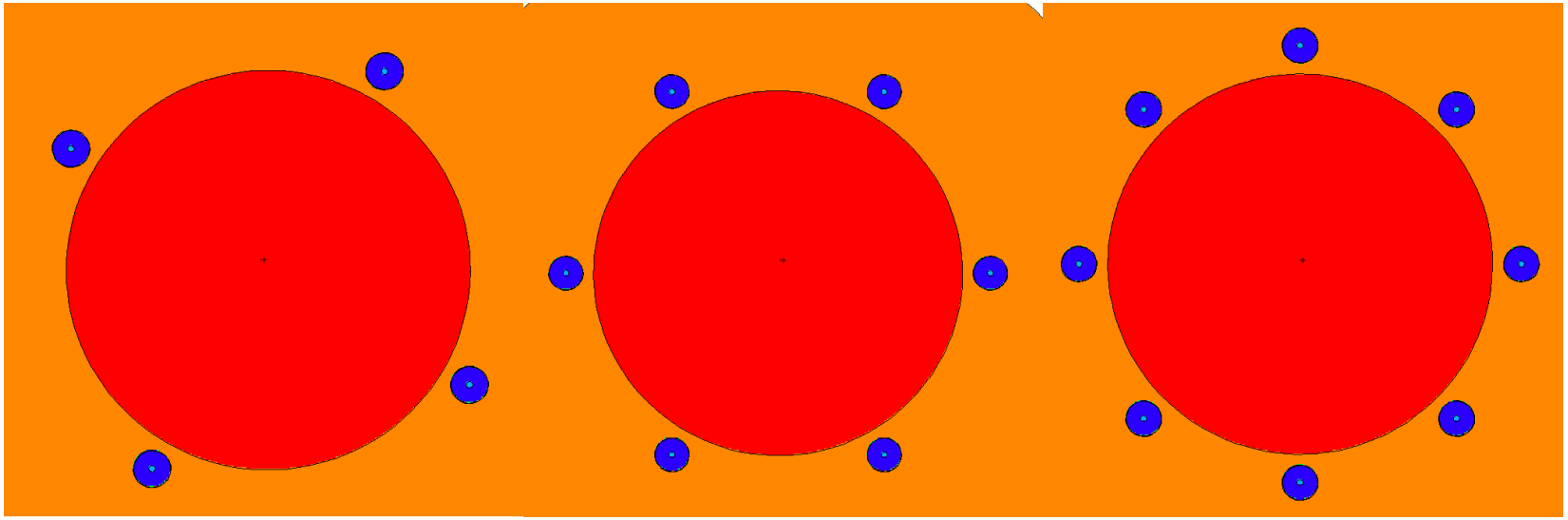
Nesimetrična razporeditev – nehomogenost

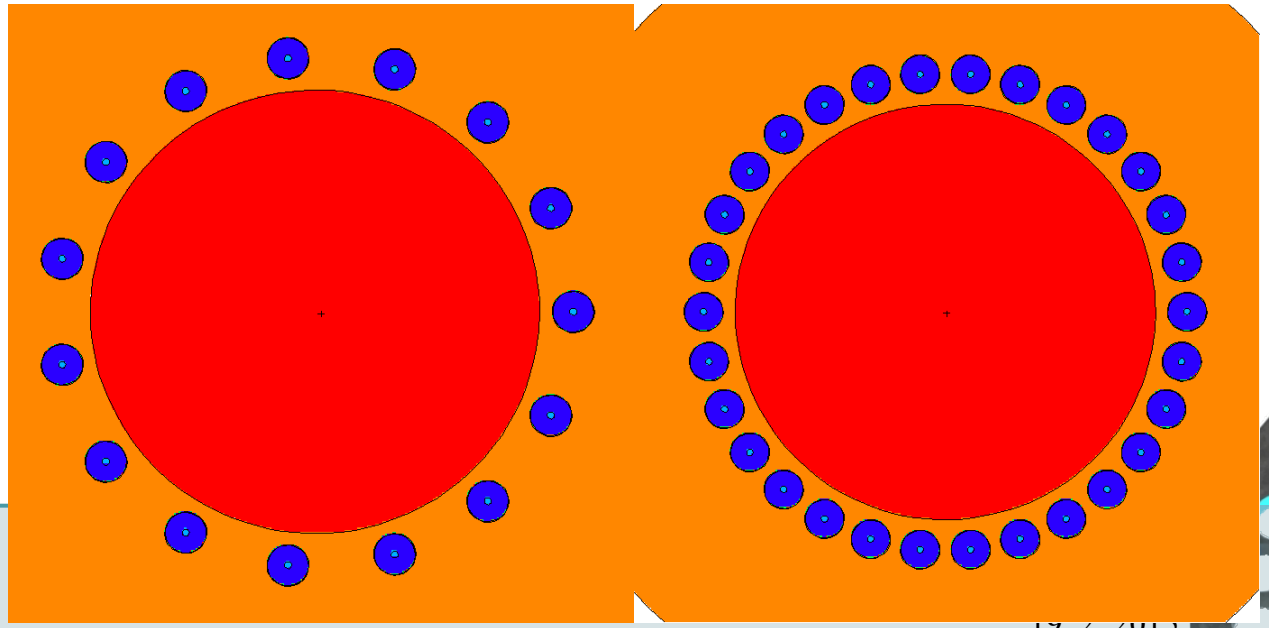


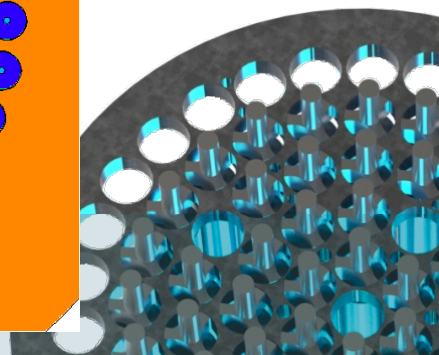




Število gorivnih elementov







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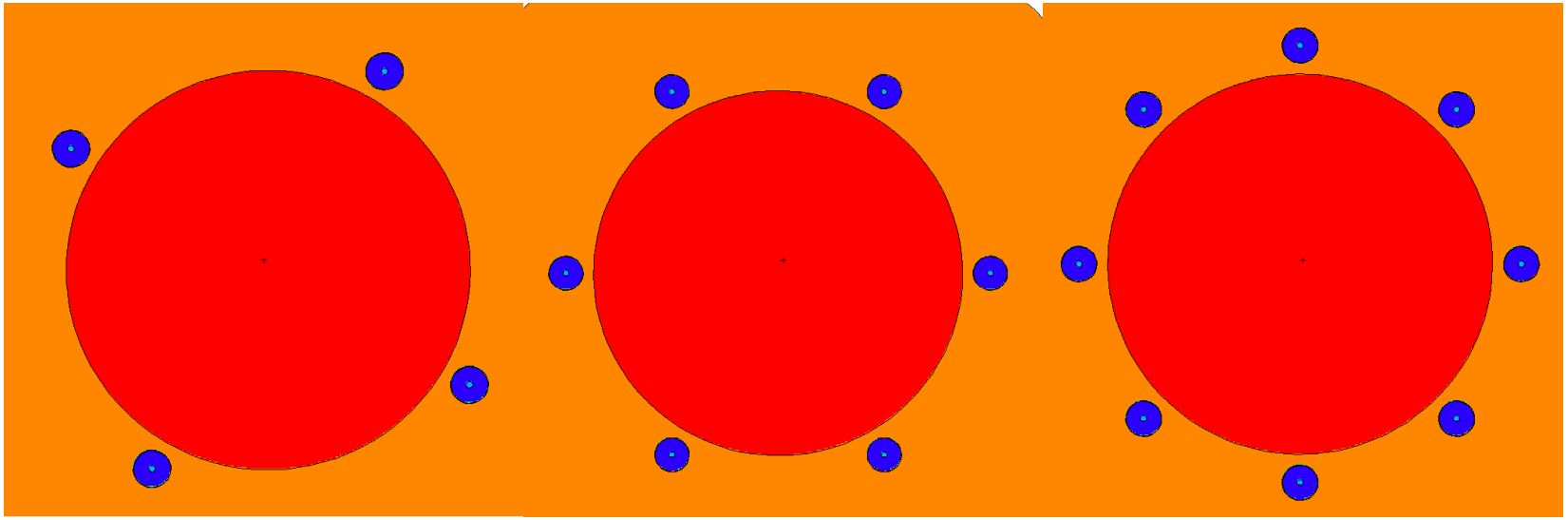
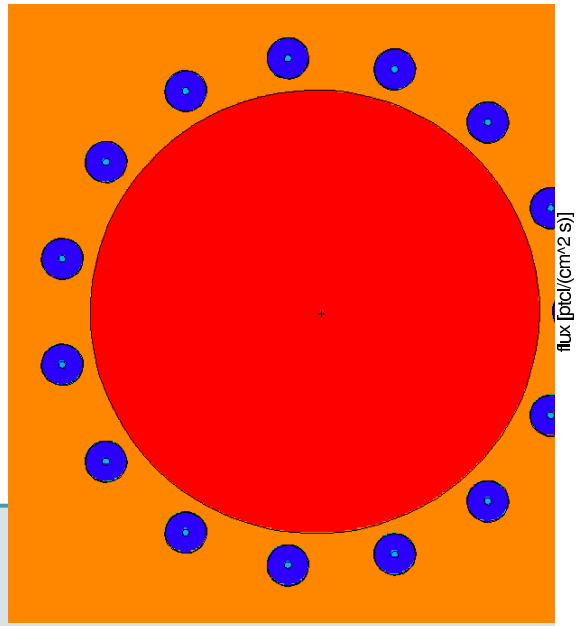
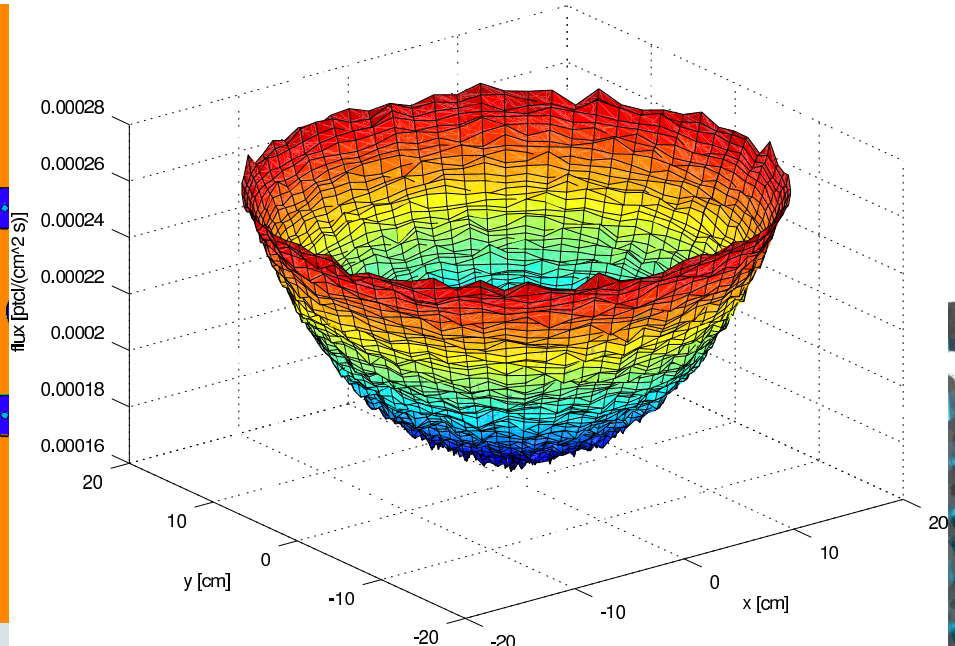
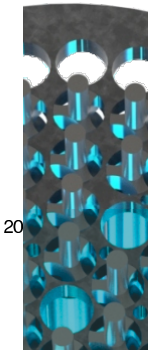
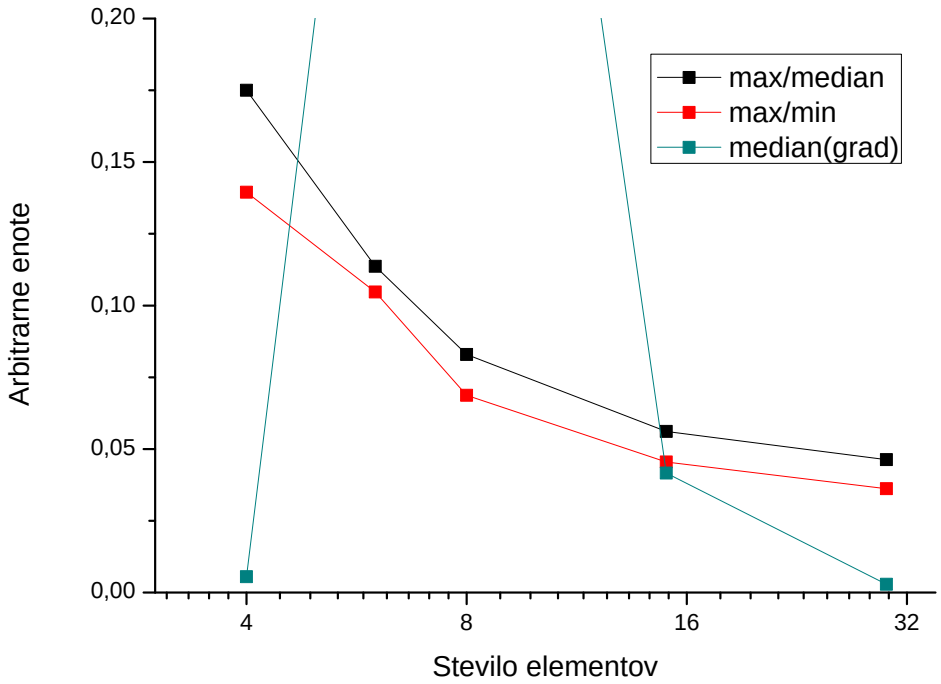


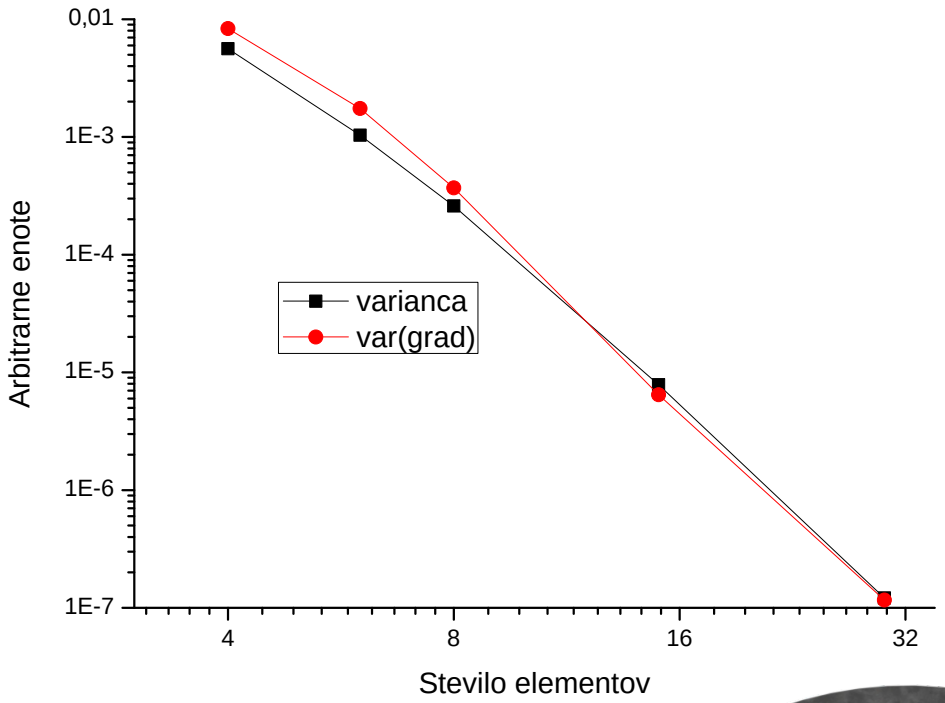




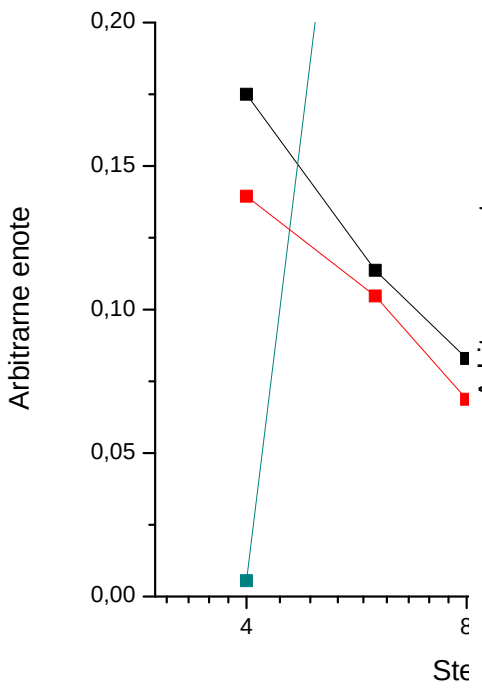


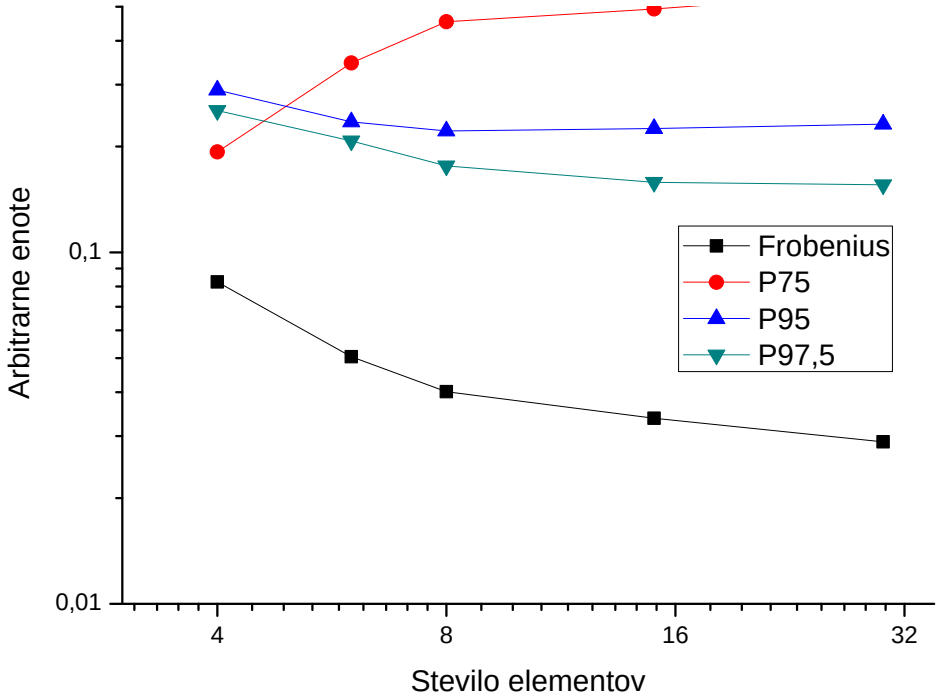
Število gorivnih elementov – nehomogenost

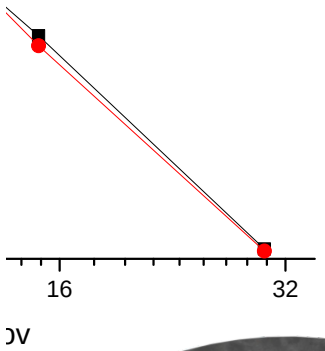


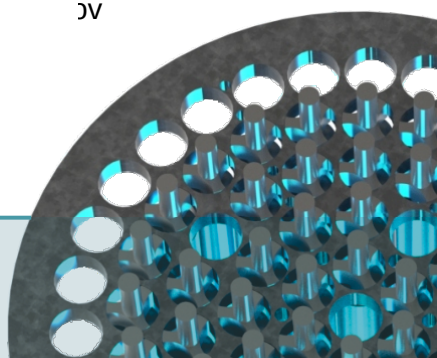


Število gorivnih elementov – nehomogenost

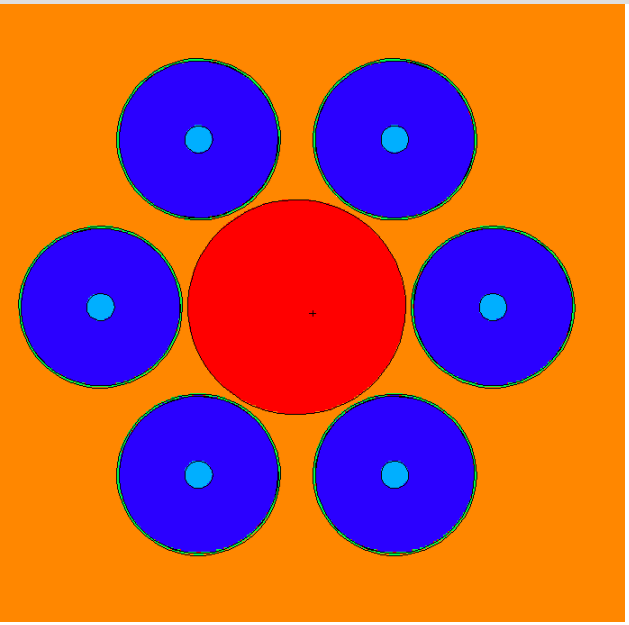




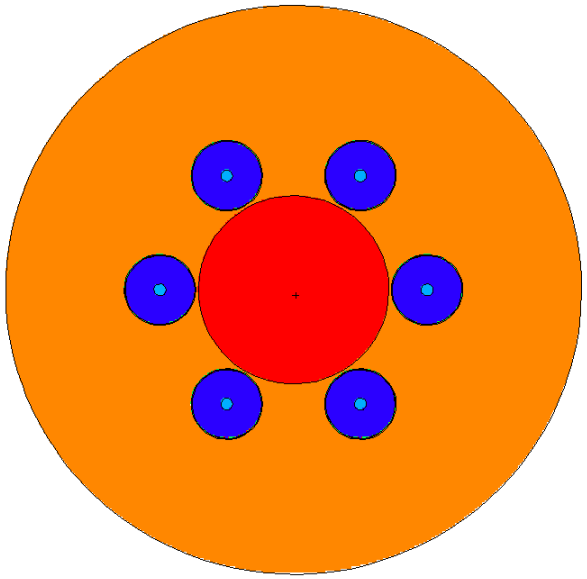




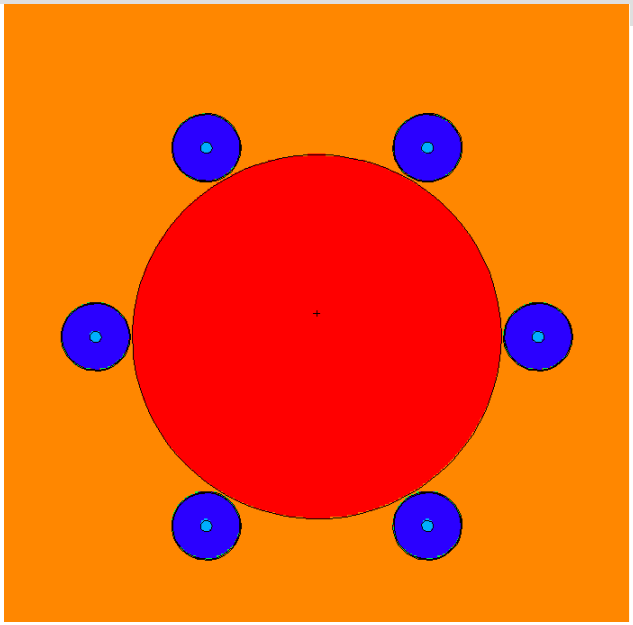
Velikost obsevalnega kanala



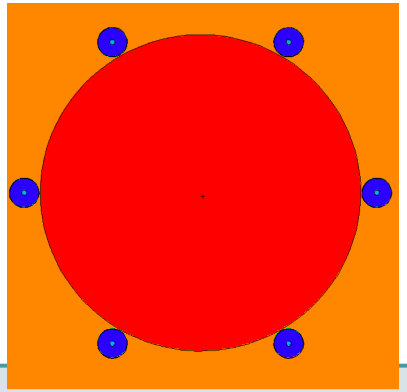
Slika: $r = 2.5\,\mathrm{cm}$



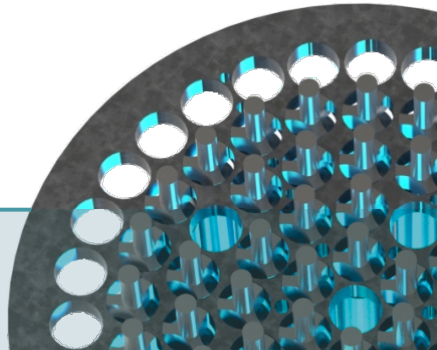
Slika: $r = 5\,\mathrm{cm}$



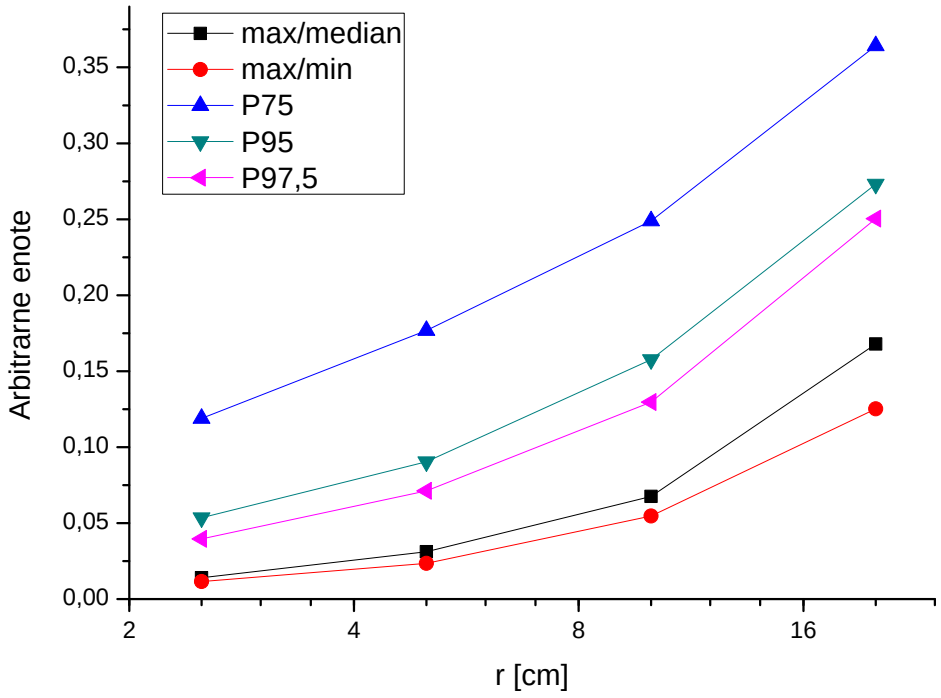
Slika: $r = 10\,\mathrm{cm}$

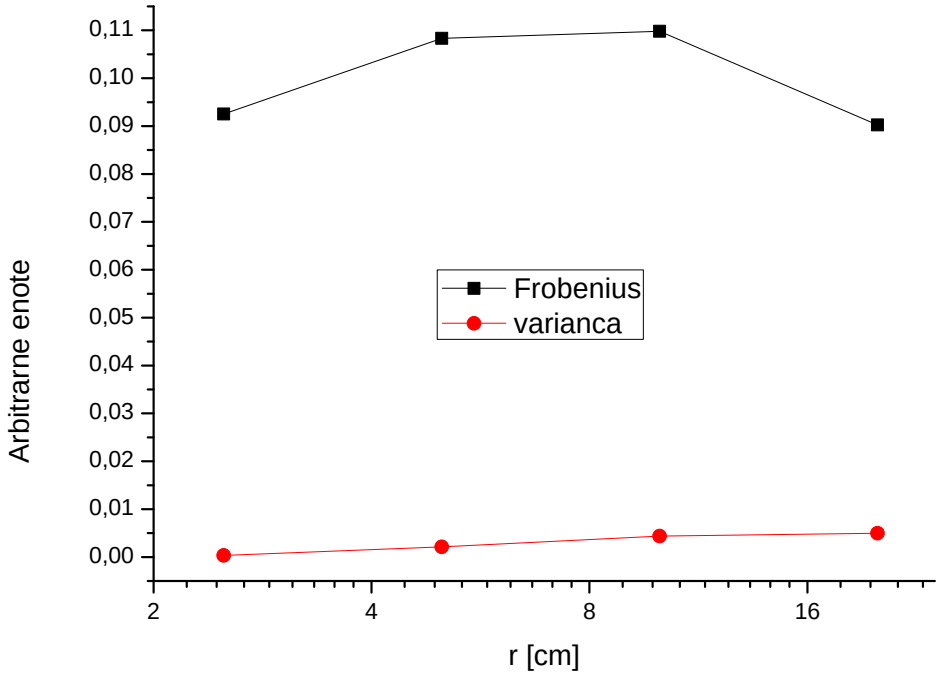


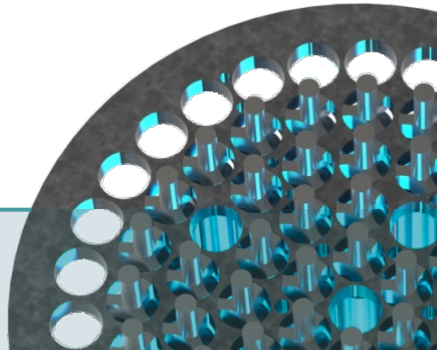
Slika: $r = 20\,\mathrm{cm}$



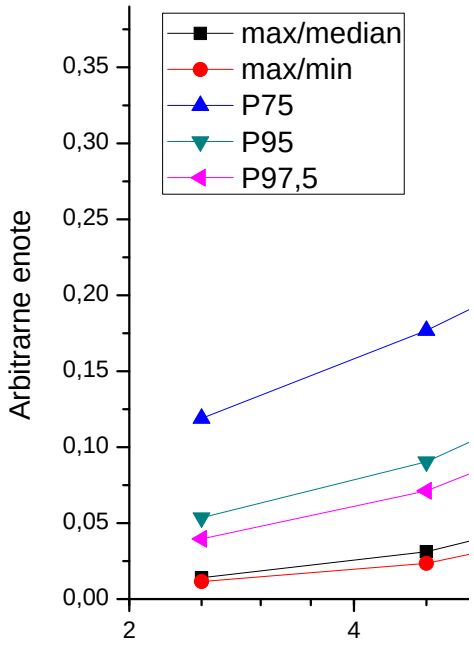
Velikost obsevalnega kanala – nehomogenost

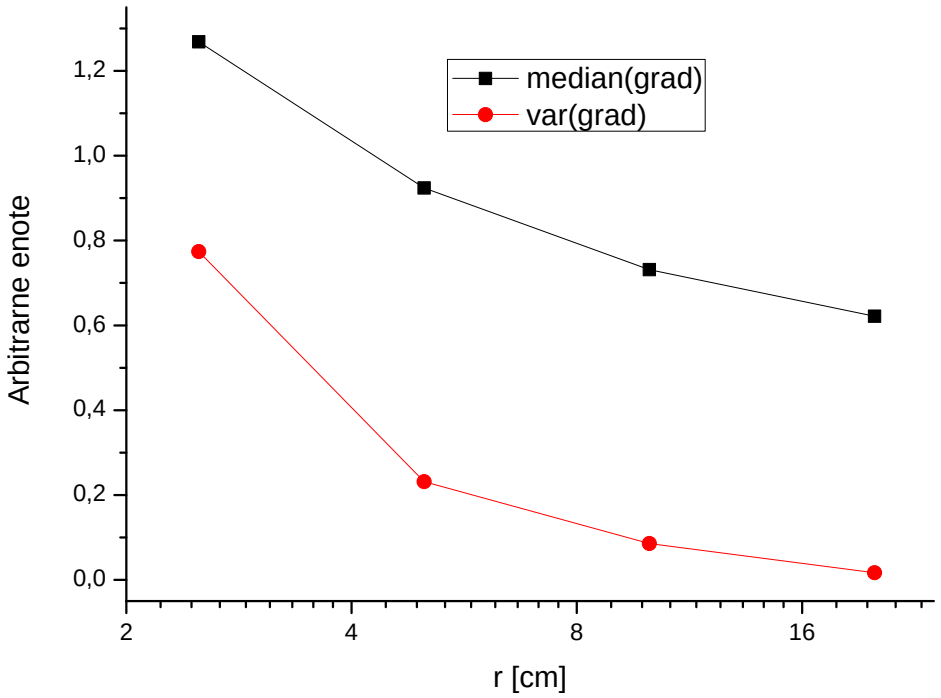


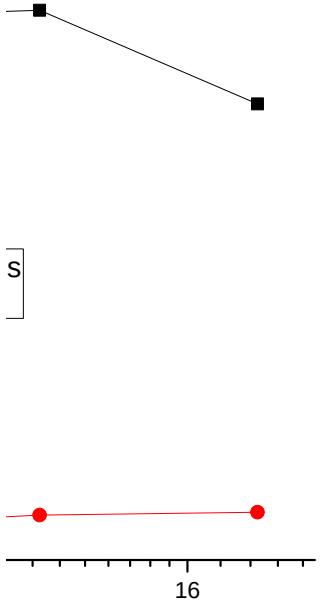


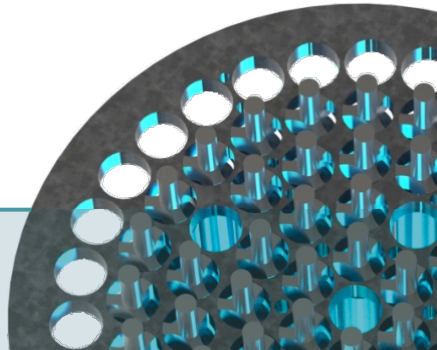


Velikost obsevalnega kanala – nehomogenost

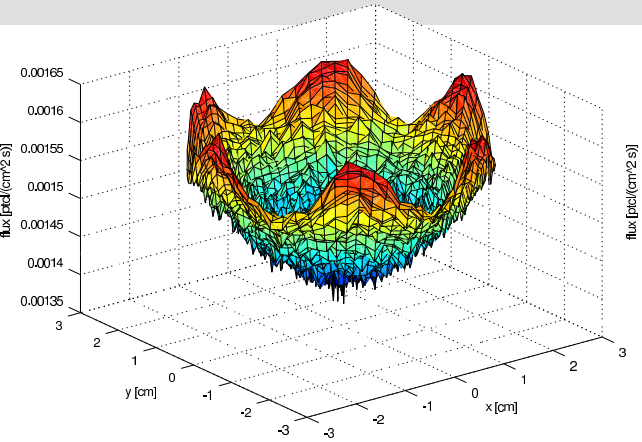




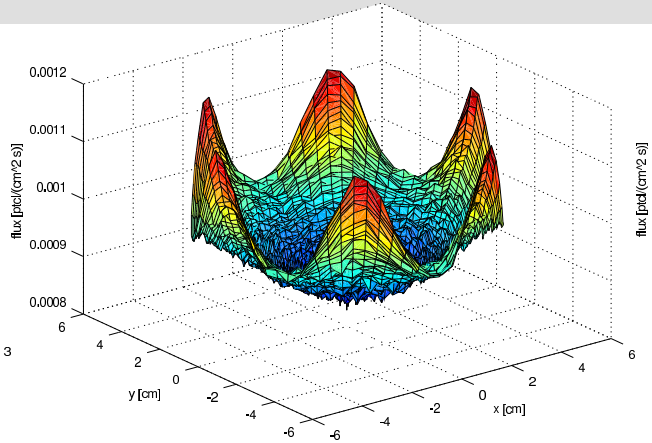




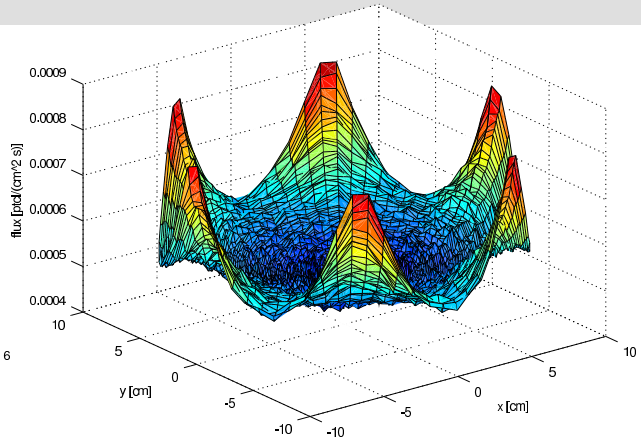
Velikost obsevalnega kanala – gradient



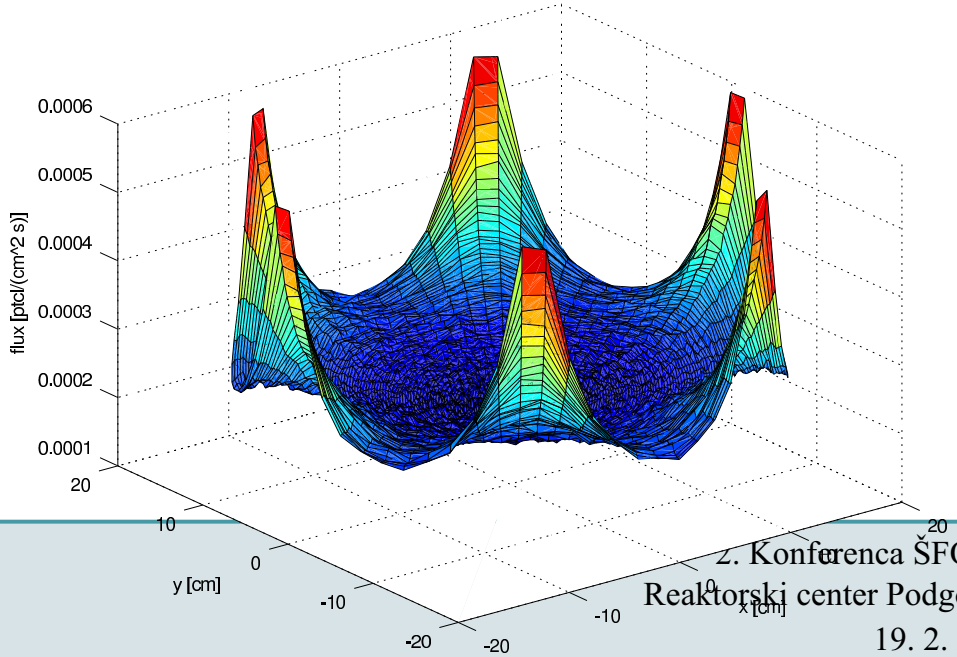
Slika: $r = 2.5\,\mathrm{cm}$



Slika: $r = 5\,\mathrm{cm}$



Slika: $r = 10\,\mathrm{cm}$



Najboljše mere homogenosti

Analiza glavnih komponent (angl. principal component analysis, PCA): prva komponenta pojasni 76 % variance

Najboljše mere homogenosti

Analiza glavnih komponent (angl. principal component analysis, PCA): prva komponenta pojasni 76 % variance

Mera	Nasičenost
max/median	0.38
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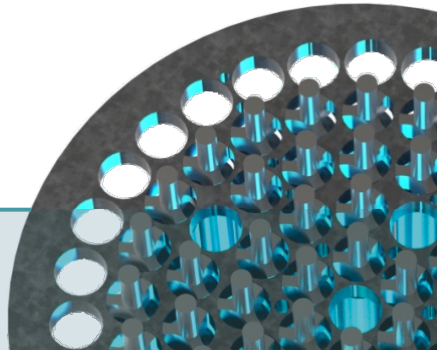
varianca	0.35
median(grad)	0.18
var(grad)	0.24
Frobenius	0.38
P_{75}	0.31
P_{95}	0.36
$P_{97.5}$	0.37



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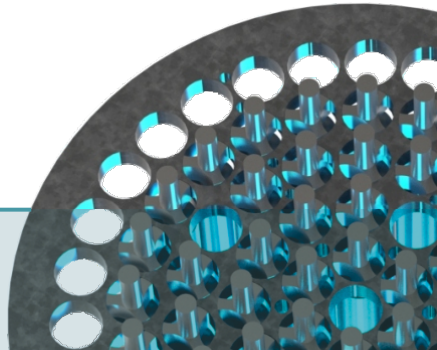
$P_{97.5}$	0.37

Multipla regresija za napoved vsote mer homogenosti



Zaključek

- Več možnosti za ovrednotenje homogenosti polja

Zaključek

- Več možnosti za ovrednotenje homogenosti polja
- Gradient občutljiv na statistične napake

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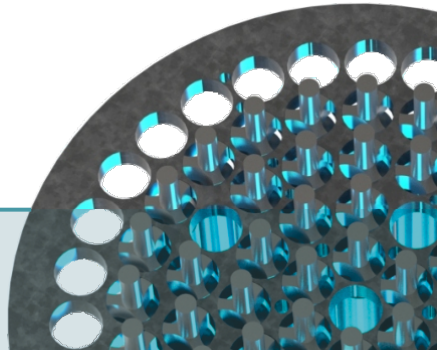
- Več možnosti za ovrednotenje homogenosti polja
- Gradient občutljiv na statistične napake
- Vsota več mer

Zaključek

- Več možnosti za ovrednotenje homogenosti polja
- Gradient občutljiv na statistične napake
- Vsota več mer
- Linearnost mere?

Hvala za pozornost!



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